Partial Vandermonde Problems and PASS Encrypt

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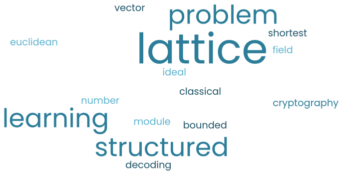
Lfant Séminaire Bordeaux, 30th November 2021



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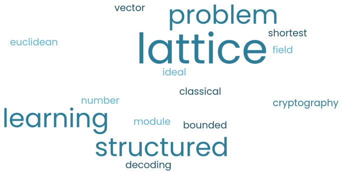

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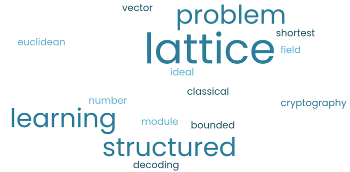

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- Codes
- Isogenies
- Multivariate Systems
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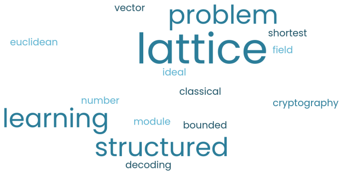

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Hard Lattice Problems

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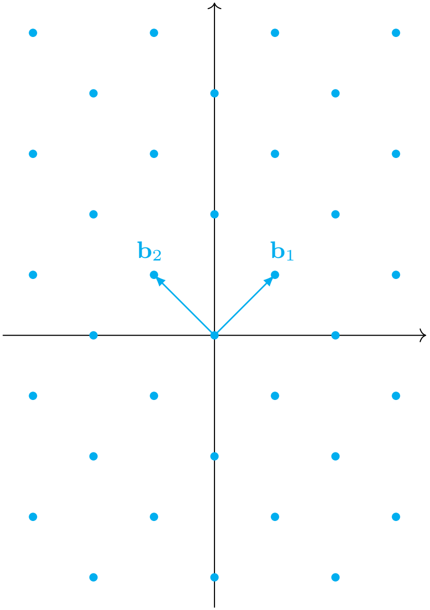

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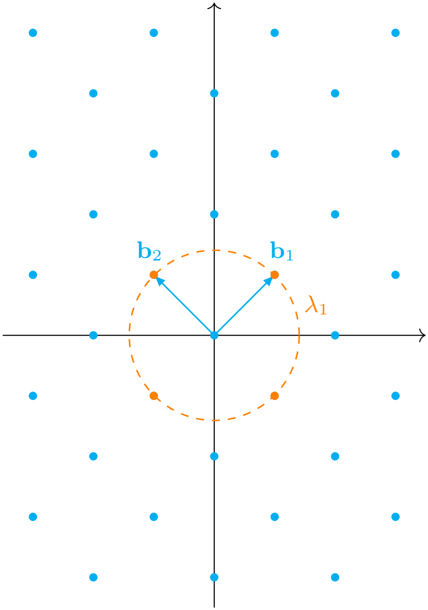
The **minimum** of Λ is

$$\lambda_1(\Lambda) := \min_{\mathbf{v} \in \Lambda \setminus \{\mathbf{0}\}} \|\mathbf{v}\|.$$

The **shortest vector problem** (SVP) asks to find a vector $\mathbf{w}$ such that $\|\mathbf{w}\| = \lambda_1(\Lambda)$.



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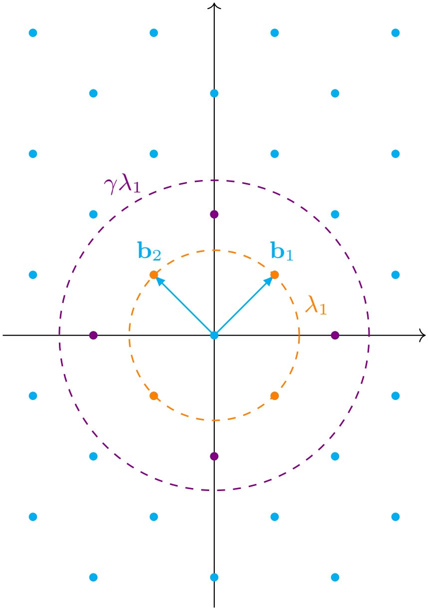

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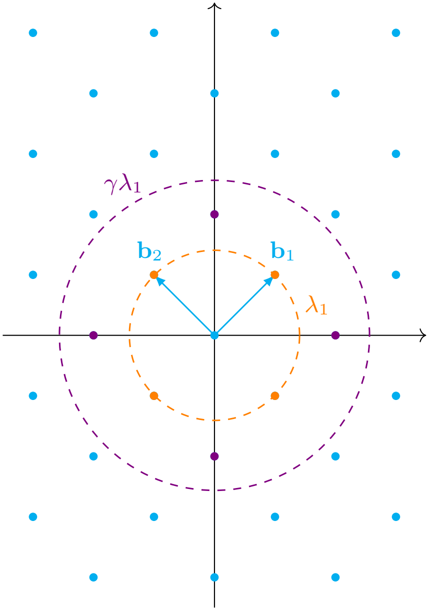

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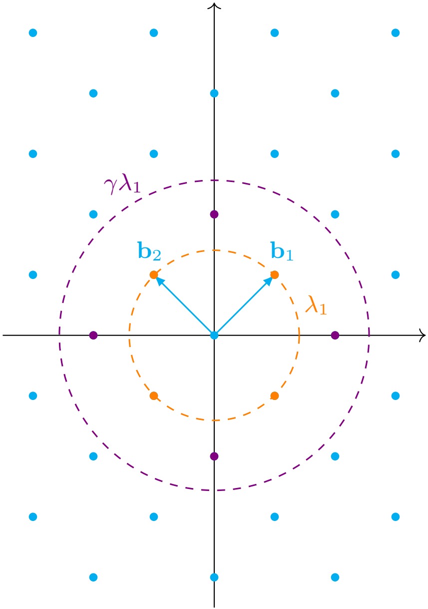
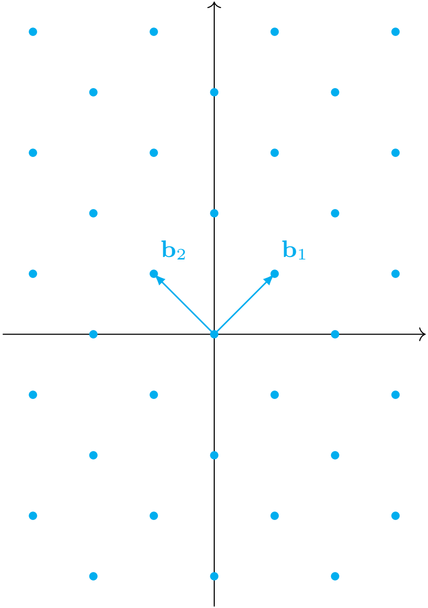
⚠ Hard to build cryptography on top of SVP_γ.



Lattice-Based Cryptography



💡 Idea: use intermediate problems!

(Main) Mathematical Problems:

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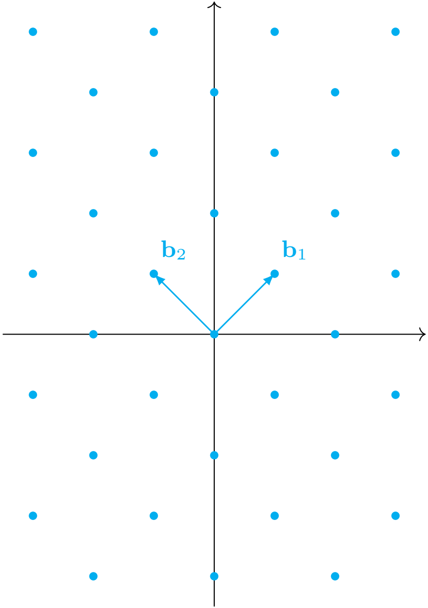

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- Partial Vandermonde Problems [HPS+14]

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NIST Competition

Started in 2016: NIST project to define new standards for post-quantum cryptography.

A majority (5 out of 7) of the finalist candidates are based on **lattice problems**.

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Observation

*Lattice-based cryptography plays a **key role** in designing post-quantum cryptography.*

Outline

Partial Vandermonde Problems

Partial Vandermonde Transform [HPS+14, LZA18]

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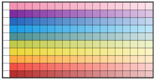

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Note: For $\Omega \subseteq \{\omega_j\}_{j=1,\ldots,n}$ write $\mathbf{V}_\Omega \cdot a = b$. **(partial Vandermonde transform)**

Partial Vandermonde Problems

Choose a random subset $\Omega \subseteq \{\omega_j\}_{j=1,\ldots,n}$ of size $|\Omega| = t$.

Partial Vandermonde knapsack problem (PV-Knap): Sample e $\sim$ DistrE over $\mathbb{Z}^n$ defining

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Search: find e

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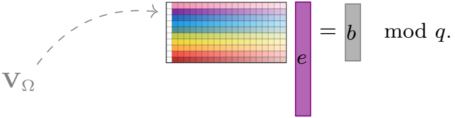

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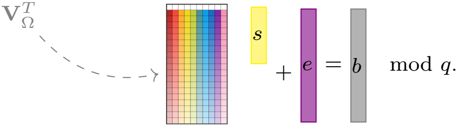
Partial Vandermonde Learning With Errors (PV-LWE): Sample s $\sim$ DistrS over $\mathbb{Z}^t$ and e $\sim$ DistrE over $\mathbb{Z}^n$ defining



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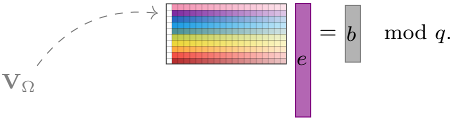

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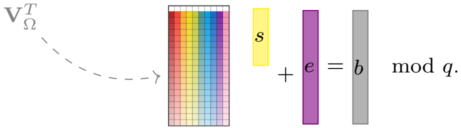

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Conjecture: Hard to solve if DistrE provides elements of small norm.

Equivalence of PV-Knap and PV-LWE

Let $t = n/2$ and set $\mathcal{P}_t = \{\Omega \subseteq \{\omega_j\}_{j=1,\ldots,n} \colon |\Omega| = t\}$.

Property 1: $\mathbf{V}_\Omega$ defines a ring homomorphism from R to $\mathbb{Z}_q^t$:

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$$\mathbf{V}_{\Omega'} \cdot \mathbf{V}_\Omega^T = 0 \in \mathbb{Z}_q^{t \times t}.$$

(parity check matrix, ⚠ only for power-of-two cyclotomics)

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Lemma (Adapted [MM11, Sec. 4.2])

Let ψ denote a distribution over $\mathbb{Z}^n \cong R$. There is an efficient reduction from PV-LWE$_\psi$ to PV-Knap$_\psi$, and vice versa.

Idea: Given $(\mathbf{V}_\Omega, b)$, with $b = \mathbf{V}_\Omega^T s + e$. Compute Ω' such that $\mathbf{V}_{\Omega'} \cdot \mathbf{V}_\Omega^T = 0$.
Then, $b' := \mathbf{V}_{\Omega'} b = \mathbf{V}_{\Omega'} e$ is an instance of PV-Knap.

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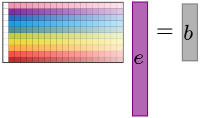

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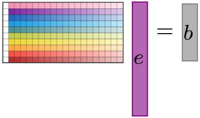

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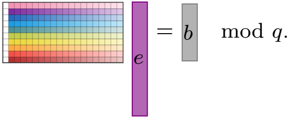
$$\begin{aligned}\Lambda_q^\perp(\mathbf{V}_\Omega) &= \{a \in R \colon \mathbf{V}_\Omega a = 0 \bmod q\} \\ &\cong I_\Omega.\end{aligned}$$

Rewrite: $\Omega \subset \{1, \ldots, n\}$ and $\langle q \rangle = \prod_{j=1}^n \mathfrak{q}_j$ and $I_\Omega = \prod_{j \in \Omega} \mathfrak{q}_j$, then $b = e \bmod I_\Omega$.

Ideal Lattice Behind Partial Vandermonde Knapsack

Choose a random subset $\Omega \subseteq \{\omega_j\}_{j=1,\dots,n}$ of size $|\Omega| = t$.

Partial Vandermonde knapsack problem (PV-Knap): Sample e $\sim$ DistrE over $\mathbb{Z}^n$ defining



Search: find e

The matrix $\mathbf{V}_\Omega$ defines an **ideal lattice**:

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Idea:

1) Solve $\mathbf{V}_\Omega y = b \bmod q$ for the unknown y in R (in general not in the support of DistrE)
2) Find a **closet vector** v of y in $\Lambda_q^{\perp}(\mathbf{V}_\Omega)$, i.e., $\|y - v\|$ smallest
3) The element $e := y - v$ is a solution to PV-Knap

⚠ Promise variant of the closest vector problem, called **Bounded Distance Decoding (BDD)**

Hard Lattice Problems

An **Euclidean lattice** Λ of rank n with a basis $\mathbf{B} = (\mathbf{b}_j)_{1 \leq j \leq n}$ is given by

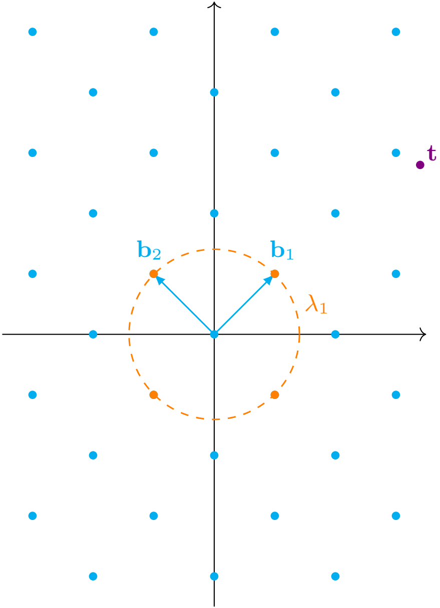
$$\Lambda(\mathbf{B}) = \left\{ \sum_{j=1}^{n} z_j \mathbf{b}_j : z_j \in \mathbb{Z} \right\}.$$

The **minimum** of Λ is

$$\lambda_1(\Lambda) := \min_{\mathbf{v} \in \Lambda \setminus \{\mathbf{0}\}} \|\mathbf{v}\|.$$

The **shortest vector problem** (SVP) asks to find a vector $\mathbf{w}$ such that $\|\mathbf{w}\| = \lambda_1(\Lambda)$.

The **closest vector problem** (CVP) asks, given a target $\mathbf{t}$, to find a closest lattice point $\mathbf{v}$ in Λ.



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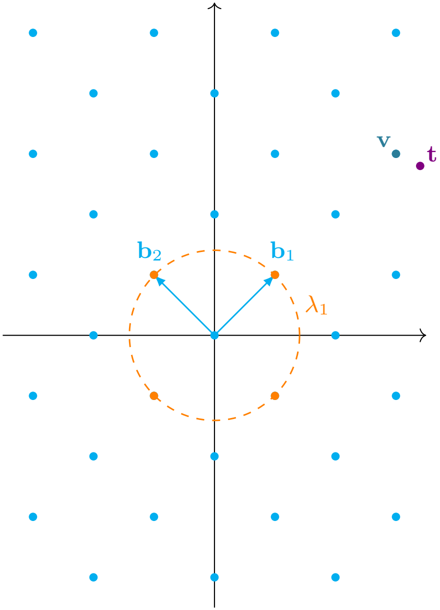
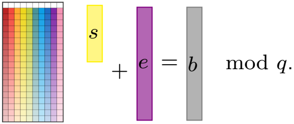


Ideal Lattice Behind Partial Vandermonde LWE

Partial Vandermonde Learning With Errors (PV-LWE): Sample s $\sim$ DistrS over $\mathbb{Z}^t$ and e $\sim$ DistrE over $\mathbb{Z}^n$ defining



Search: find e (and secret s)

This is an instance of **BDD** in the **ideal lattice**

$$\Lambda_q(\mathbf{V}_\Omega) = \{a \in R \colon a = \mathbf{V}_\Omega^T s \bmod q \text{ for some } s \in \mathbb{Z}_q^t\}$$

Recall Property 3: it exists Ω' such that $\mathbf{V}_{\Omega'} \cdot \mathbf{V}_\Omega^T = 0$. It yields $\Lambda_q(\mathbf{V}_\Omega) \subseteq I_{\Omega'}$ and $\mathcal{N}(I_{\Omega'}) = n - t = \mathcal{N}(\Lambda_q(\mathbf{V}_\Omega))$, thus isomorph.

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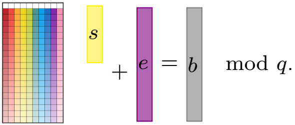

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PASS Encrypt

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$$\mathbf{V}^{-1}(e_1 || c') = \mathbf{V}^{-1}(\mathbf{V}(f \cdot r' + s')) = f \cdot pr + ps + m = m \bmod p$$

if f, r and s are small enough

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$\Rightarrow$ leaky variant of **PV-Knap**, that we call the **PASS problem**.

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⚠ PASS problem is tailored to PASS Encrypt!
? Reduce it from some more general problem?

Properties of PASS Encrypt

Homomorphic properties:

Addition: $\mathsf{Enc}(\mathsf{pk}, m_1) + \mathsf{Enc}(\mathsf{pk}, m_2) = \mathsf{Enc}(\mathsf{pk}, m_1 + m_2)$

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Efficiency:

Scheme	NTRU [HPS98]	P-LWE Regev [LP11]	PASS Encrypt
$\frac{\lvert c\rvert + \lvert\mathsf{pk}\rvert}{\lvert m\rvert}$	$2\log_2 q$	$3\log_2 q$	$2.5\log_2 q$

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Concrete Security:

Known: key recovery and randomness recovery attacks [HS15, DHSS20]

New: plaintext recovery using hints attacks

make use of leaky LWE estimator of Dachman-Soled et al. [DDGR20]

Conclusion and Perspectives

Open Questions and Perspectives

Follow-ups

- Construct encryption scheme based only on PV-LWE / PV-Knap

Questions **?**

- Hardness of partial Vandermonde problems
  - Cryptanalysis?
  - Worst-case to average-case reductions as for LWE?
- More cryptographic applications

Thank you.

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