Practical Fully Secure Inner Product Functional Encryption modulo p

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Functional Encryption (FE)

Traditional Encryption: All or Nothing

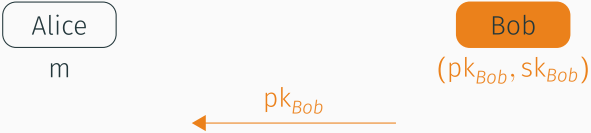
Alice

m

Bob

(pk_{Bob}, sk_{Bob})

Traditional Encryption: All or Nothing



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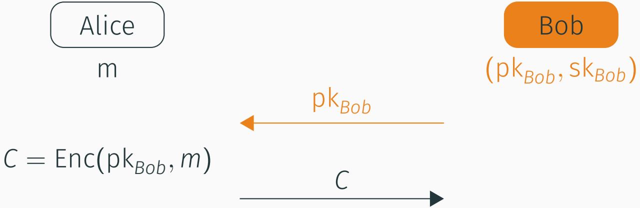

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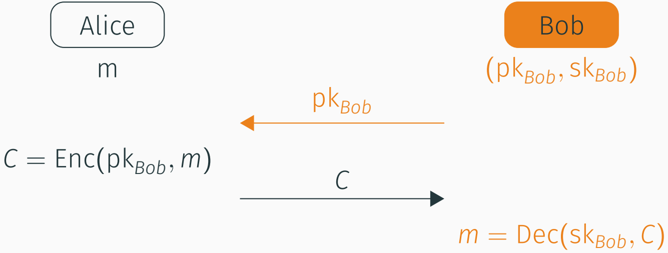
$C = \mathrm{Enc}(\mathrm{pk}_{Bob}, m)$

C

Traditional Encryption: All or Nothing



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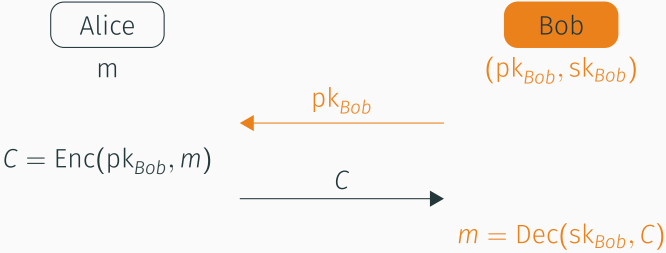

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pk_{Bob}

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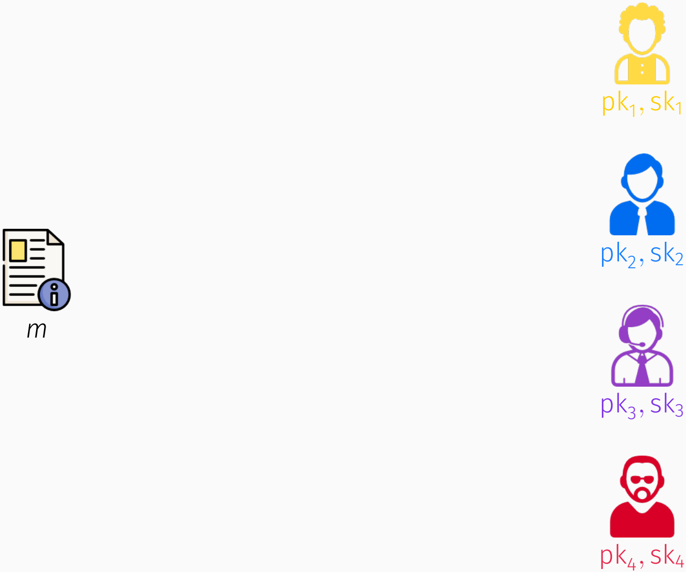
Bob gets **all** the information in *m*.

Fine Grained Access to Info with Traditional Encryption



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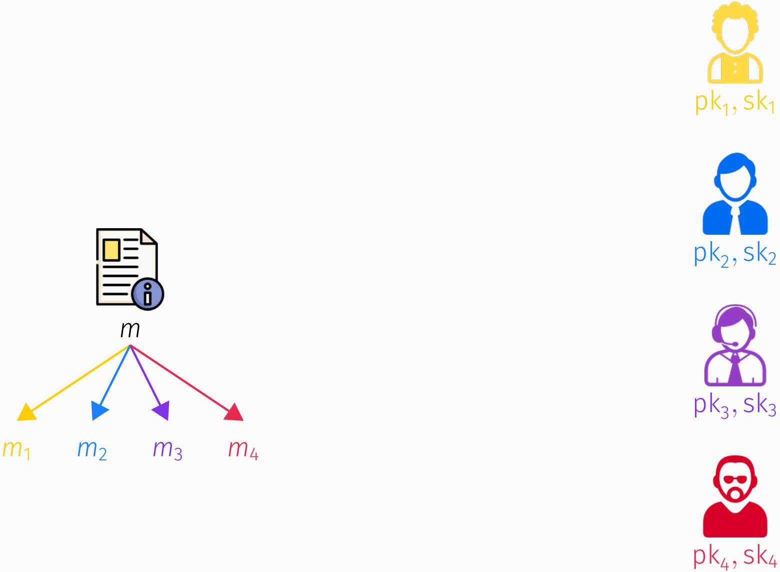

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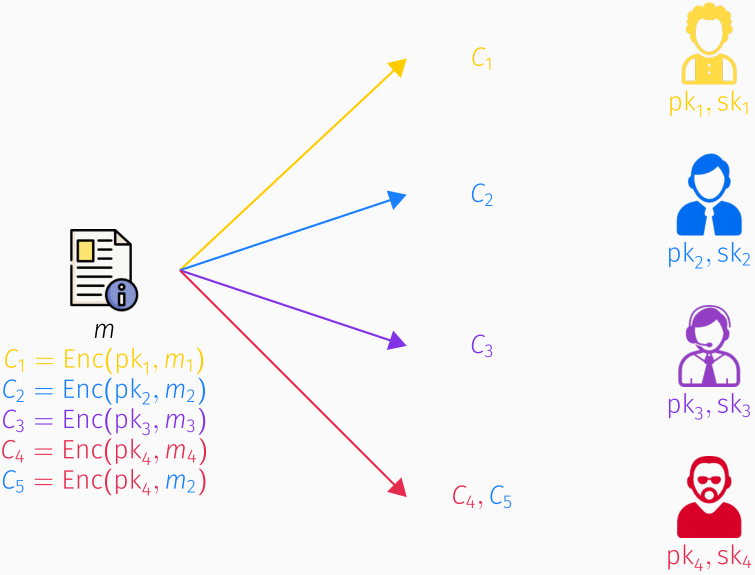


m

$C_1 = \mathsf{Enc}(\mathsf{pk}_1, m_1)$
$C_2 = \mathsf{Enc}(\mathsf{pk}_2, m_2)$
$C_3 = \mathsf{Enc}(\mathsf{pk}_3, m_3)$
$C_4 = \mathsf{Enc}(\mathsf{pk}_4, m_4)$
$C_5 = \mathsf{Enc}(\mathsf{pk}_4, m_2)$

C_1

C_2

C_3

C_4, C_5

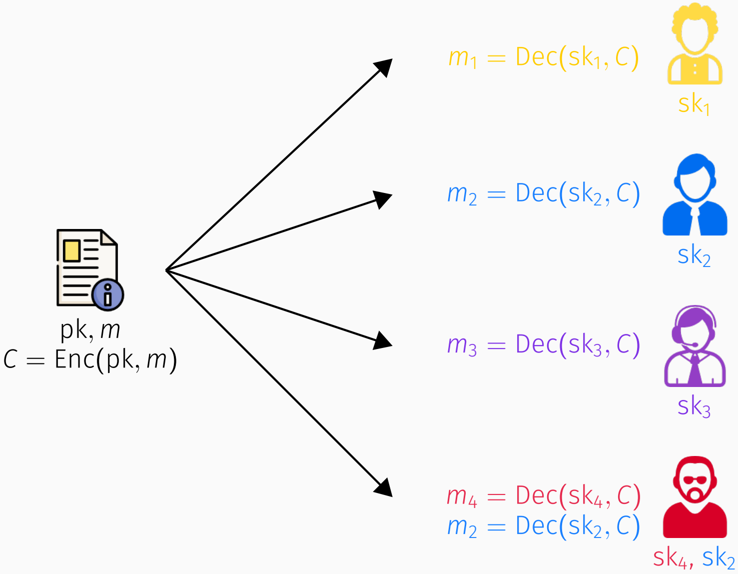
$\mathsf{pk}_1, \mathsf{sk}_1$

$\mathsf{pk}_2, \mathsf{sk}_2$

$\mathsf{pk}_3, \mathsf{sk}_3$

$\mathsf{pk}_4, \mathsf{sk}_4$

Ideal Fine Grained Access to Information



pk, m

$C = \mathsf{Enc}(\mathsf{pk}, m)$

$m_1 = \mathsf{Dec}(\mathsf{sk}_1, C)$

sk_1

$m_2 = \mathsf{Dec}(\mathsf{sk}_2, C)$

sk_2

$m_3 = \mathsf{Dec}(\mathsf{sk}_3, C)$

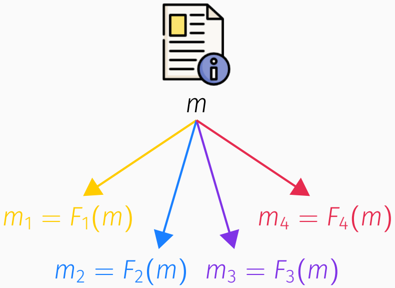
sk_3

$m_4 = \mathsf{Dec}(\mathsf{sk}_4, C)$

$m_2 = \mathsf{Dec}(\mathsf{sk}_2, C)$

sk_4, sk_2

Functional Encryption



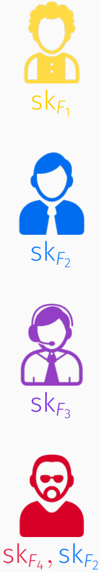
m

$m_1 = F_1(m)$

$m_2 = F_2(m)$

$m_3 = F_3(m)$

$m_4 = F_4(m)$



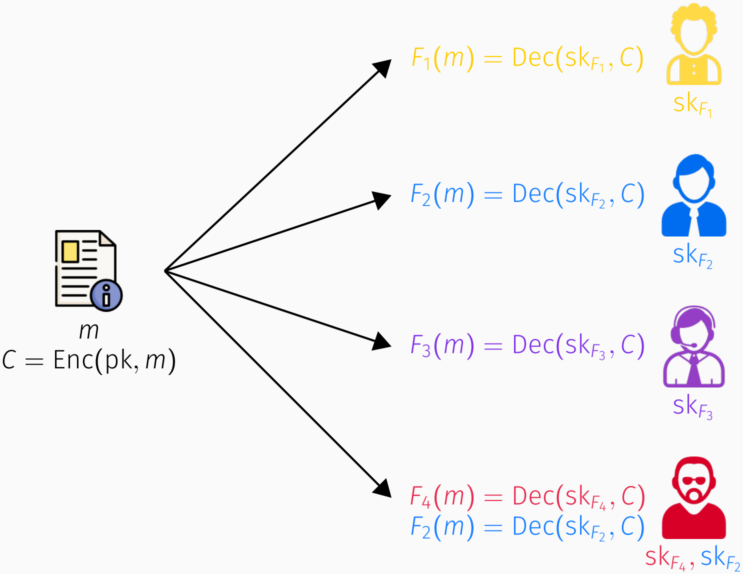
sk_{F_1}

sk_{F_2}

sk_{F_3}

$\mathsf{sk}_{F_4}, \mathsf{sk}_{F_2}$

Functional Encryption



m

$C = \mathrm{Enc}(\mathrm{pk}, m)$

$F_1(m) = \mathrm{Dec}(\mathrm{sk}_{F_1}, C)$

sk_{F_1}

$F_2(m) = \mathrm{Dec}(\mathrm{sk}_{F_2}, C)$

sk_{F_2}

$F_3(m) = \mathrm{Dec}(\mathrm{sk}_{F_3}, C)$

sk_{F_3}

$F_4(m) = \mathrm{Dec}(\mathrm{sk}_{F_4}, C)$

$F_2(m) = \mathrm{Dec}(\mathrm{sk}_{F_2}, C)$

$\mathrm{sk}_{F_4}, \mathrm{sk}_{F_2}$

Application: Spam filtering for encrypted emails

Recipient

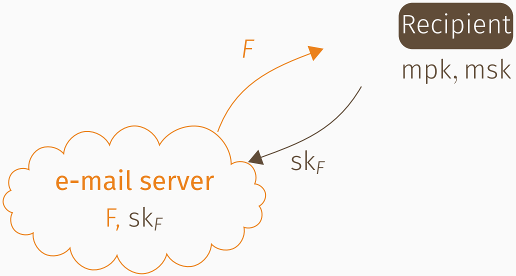
mpk, msk

e-mail server

F

$$F(m) = \begin{cases} 1 & \text{if m is spam} \\ 0 & \text{otherwise} \end{cases}$$

Application: Spam filtering for encrypted emails



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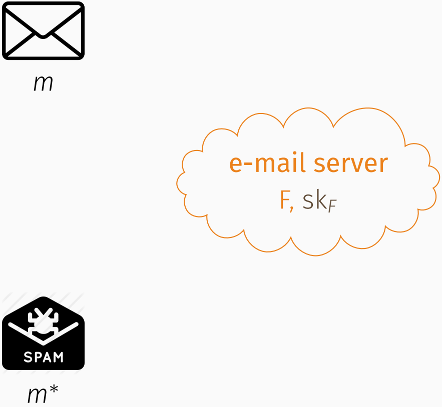


m

e-mail server

F, sk_F

m^*

Application: Spam filtering for encrypted emails

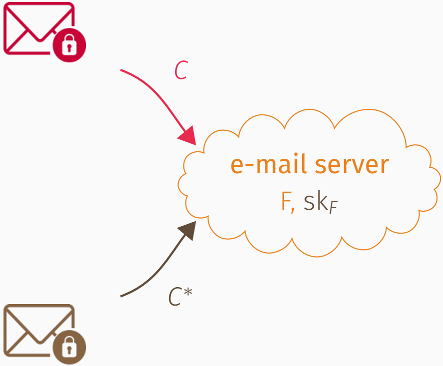
$C = \mathsf{Enc}(\mathsf{mpk}, m)$

e-mail server

F, sk_F

$C^* = \mathsf{Enc}(\mathsf{mpk}, m^*)$

Application: Spam filtering for encrypted emails



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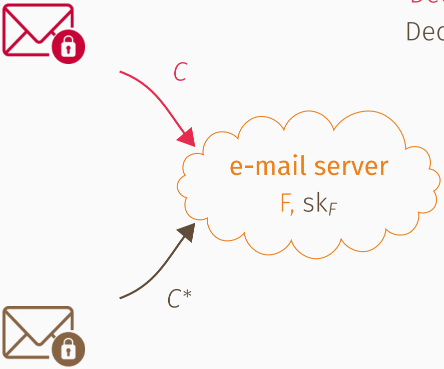
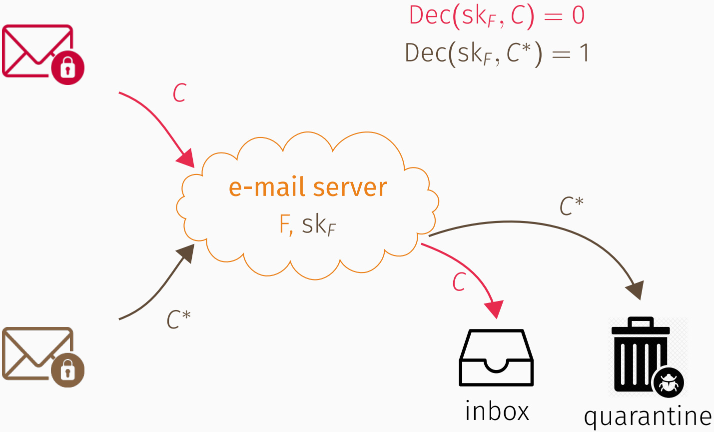


$\mathsf{Dec}(\mathsf{sk}_F, C) = 0$

$\mathsf{Dec}(\mathsf{sk}_F, C^*) = 1$

Application: Spam filtering for encrypted emails

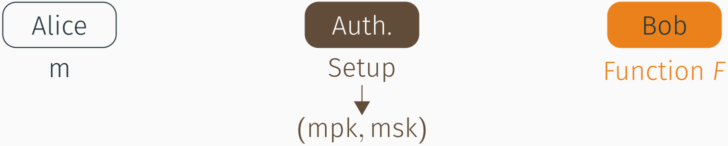


$\mathsf{Dec}(sk_F, C) = 0$

$\mathsf{Dec}(sk_F, C^*) = 1$

e-mail server learns **one bit of information**

Functional Encryption [BSW11]



Alice

m

Auth.

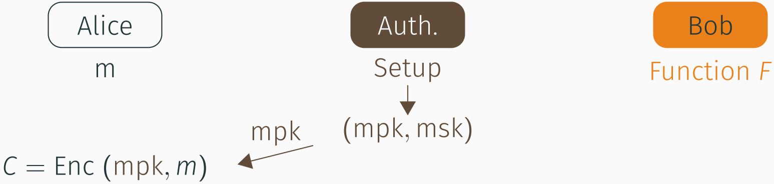
Setup

(mpk, msk)

Bob

Function *F*

Functional Encryption [BSW11]



Alice

m

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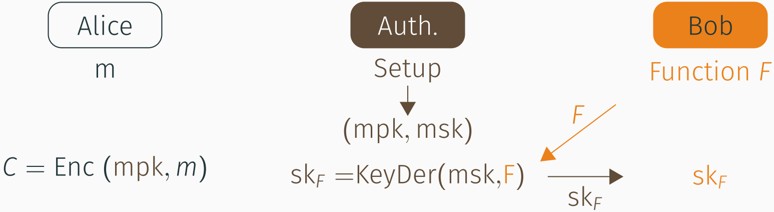
mpk

$C = \mathrm{Enc}\left(\mathrm{mpk}, m\right)$

Bob

Function F

Functional Encryption [BSW11]



Alice

m

$C = \text{Enc}\left(\text{mpk}, m\right)$

Auth.

Setup

(mpk, msk)

$\text{sk}_F = \text{KeyDer}(\text{msk}, F)$ $\xrightarrow[\text{sk}_F]{}$

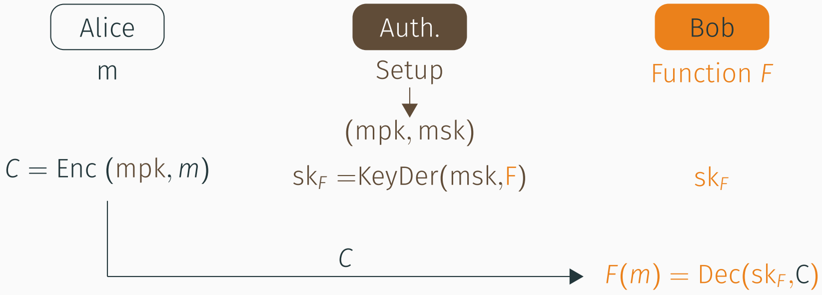
Bob

Function F

F

sk_F

Functional Encryption [BSW11]



Alice

m

Auth.

Setup

(mpk, msk)

Bob

Function F

$C = \mathrm{Enc}\,(\mathrm{mpk}, m)$

$\mathrm{sk}_F = \mathrm{KeyDer}(\mathrm{msk}, F)$

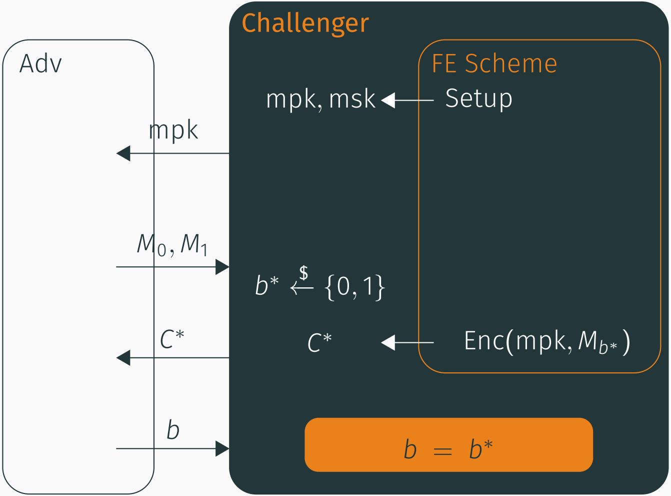
sk_F

C

$F(m) = \mathrm{Dec}(\mathrm{sk}_F, \mathrm{C})$

Bob **only learns** $F(m)$.

FE Security – Indistinguishability



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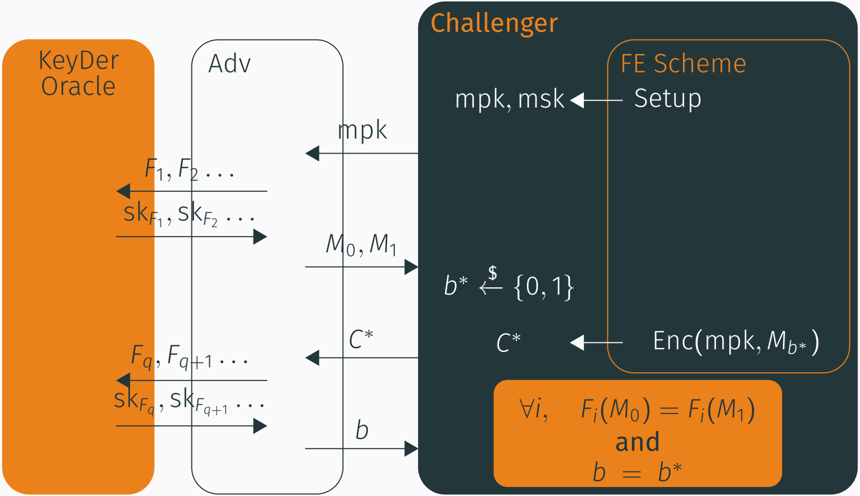

KeyDer Oracle

Adv

Challenger

FE Scheme

mpk, msk ← Setup

mpk

$F_1, F_2 \ldots$

$\mathsf{sk}_{F_1}, \mathsf{sk}_{F_2} \ldots$

M_0, M_1

$b^* \xleftarrow{\$} \{0, 1\}$

C^*

$C^* \leftarrow \mathsf{Enc}(\mathsf{mpk}, M_{b^*})$

$F_q, F_{q+1} \ldots$

$\mathsf{sk}_{F_q}, \mathsf{sk}_{F_{q+1}} \ldots$

b

$\forall i, \quad F_i(M_0) = F_i(M_1)$

and

$b = b^*$

Limits of General Functional Encryption

We don’t know how to build **practical** FE for general functions

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$\Rightarrow$ **Linear Functions:** simple with many applications

Limits of General Functional Encryption

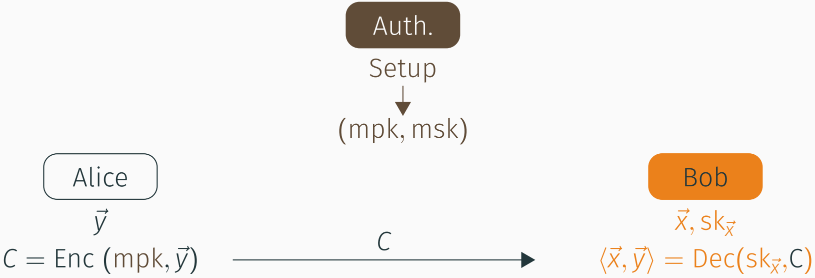
We **don't know** how to build **practical** FE for **general functions**

$\Rightarrow$ **Linear Functions: simple** with **many applications**

- Understand general FE
- Statistical analysis on encrypted data
- Evaluation of polynomials over encrypted data
- Constructing trace-and-revoke system
- etc.

The Inner Product Functionality

The inner product functionality



Auth.

Setup

$\downarrow$

$(\mathrm{mpk}, \mathrm{msk})$

Alice

$\vec{y}$

$C = \mathrm{Enc}\left(\mathrm{mpk}, \vec{y}\right)$

$\xrightarrow{C}$

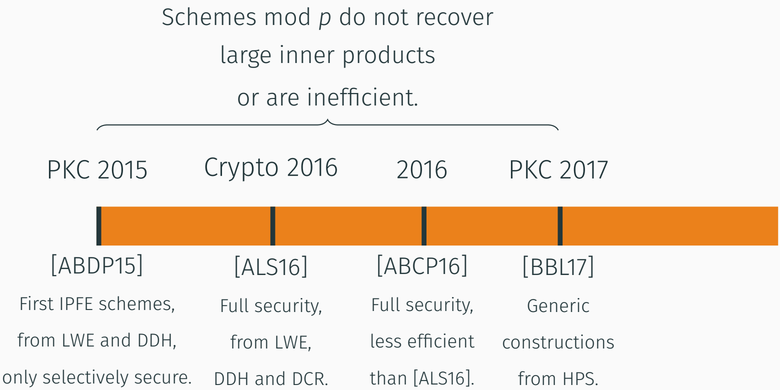
Bob

$\vec{x}, \mathrm{sk}_{\vec{x}}$

$\langle \vec{x}, \vec{y} \rangle = \mathrm{Dec}(\mathrm{sk}_{\vec{x}}, \mathrm{C})$

$$\begin{aligned} F_x : \mathcal{R}^{\ell} &\mapsto \mathcal{R} \\ y &\mapsto \langle \vec{x}, \vec{y} \rangle \end{aligned}$$

Previous work



Schemes mod p do not recover
large inner products
or are inefficient.

PKC 2015

[ABDP15]

First IPFE schemes,
from LWE and DDH,
only selectively secure.

Crypto 2016

[ALS16]

Full security,
from LWE,
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PKC 2017

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Generic
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from HPS.

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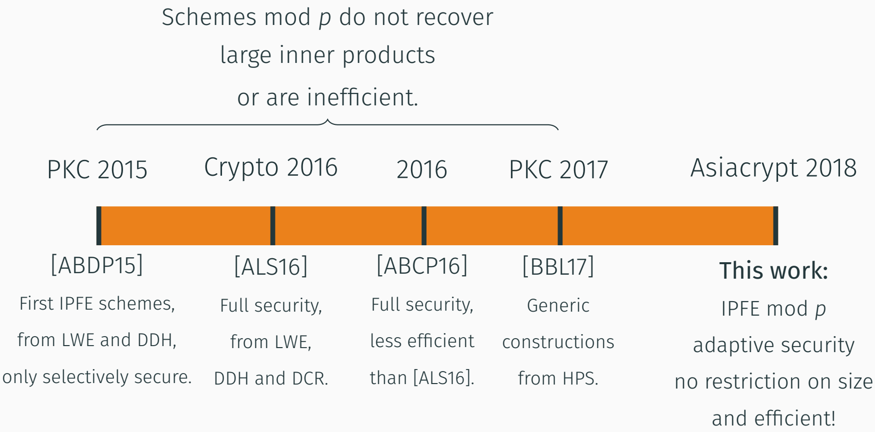

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Generic constructions from HPS.

Asiacrypt 2018

This work:

IPFE mod p adaptive security no restriction on size and efficient!

The Hard Subgroup Membership (HSM) Assumption

Framework (sketch) [CL15]

Group with an easy discrete logarithm (DL) subgroup

- $G = \langle g \rangle$ cyclic group of order $p \cdot s$ such that $\gcd(p, s) = 1$.
- p large prime
- $F = \langle f \rangle$ subgroup of G of order p.
- $G^p = \langle g_p \rangle = \{x^p, x \in G\}$ subgroup of G of order s,

$$G = F \times G^p.$$

- DL is **easy** in F (DL: given f and $h = f^x$, find $x \in \mathbf{Z}/p\mathbf{Z}$)

New Assumption

Hard Subgroup Membership problem HSM:

Hard to distinguish p-th powers in G

$$\{x \xleftarrow{\$} G\} \approx_c \{x \xleftarrow{\$} G^p\}.$$

Instantiation in class groups of an imaginary quadratic field

- $K = \mathbb{Q}(\sqrt{\Delta_K})$, $\Delta_K < 0$ and $\Delta_K \equiv 1 \mod 4$

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- $\phi_p : C(\mathcal{O}_{\Delta_p}) \mapsto C(\mathcal{O}_{\Delta_K})$ **surjection** where $\mathrm{Ker}(\phi_\mathrm{p})$ of order p.
  - Implies $h(\mathcal{O}_{\Delta_p}) = p \times h(\mathcal{O}_{\Delta_K})$

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    - Implies $h(\mathcal{O}_{\Delta_p}) = p \times h(\mathcal{O}_{\Delta_K})$
- $\mathfrak{a}$ ideal of $\mathcal{O}_\Delta$ can be written as $\mathfrak{a} = (a\mathbf{Z} + \frac{-b+\sqrt{\Delta}}{2}\mathbf{Z})$ and represented by (a, b); for $a \in \mathbf{N}, b \in \mathbf{Z}, b^2 \equiv \Delta \mod 4a$

Instantiation in class groups of an imaginary quadratic field

- $\mathfrak{t} = (p^2, p) \in \mathcal{O}_{\Delta_p}$, set $f = [\mathfrak{t}]$
  $\Rightarrow f$ generates $\mathrm{Ker}(\phi_{\mathrm{p}})$ (subgroup of order p of $C(\mathcal{O}_{\Delta_p})$), and

$$f^m = \left[p^2\mathbf{Z} + \frac{-L(m)p + \sqrt{\Delta_p}}{2}\mathbf{Z} \right]$$

$L(m)$: odd integer in $[-p, p]$ s.t. $L(m) = 1/m \mod p$

$F = <f>$ **cyclic group of order** p**, and** DL **easy**

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  $L(m)$: odd integer in $[-p, p]$ s.t. $L(m) = 1/m \mod p$
  $F = < f >$ cyclic group of order p, and DL easy
- To build G^p:
  - $\hat{g} \xleftarrow{\$} C(\mathcal{O}_{\Delta_K})$ of order $s|h(\mathcal{O}_{\Delta_K})$.
  - $\gcd(p, h(\mathcal{O}_{\Delta_K})) = 1 \Rightarrow \gcd(p, s) = 1$
  - $g_p = (\phi_p^{-1}(\hat{g}))^p \in C(\mathcal{O}_{\Delta_p})$

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  - $g_p = (\phi_p^{-1}(\hat{g}))^p \in C(\mathcal{O}_{\Delta_p})$
- Set $g = g_p \cdot f$ and $G =< g >$ of order ps

Security in class groups of an imaginary quadratic field

- Security from **hardness of class number computation** and **DL problem in $C(\mathcal{O}_{\Delta_K})$**.
- Best known algos use index calculus method

  $\Rightarrow L(1/2)$ complexity
- **Shorter keys!**

	$\lambda = 112$		$\lambda = 128$	
size	this work	DCR	this work	DCR
$(p, \tilde{s})$	$(112, 684)$	$(1024, 2046)$	$(128, 924)$	$(1536, 3070)$
el^t of G	1572	4096	2084	6144
secret key	$112(\ell+1)+684$	$2048(\ell+2)$	$128(\ell+1)+924$	$3072(\ell+2)$

Sampling exponents

Problem

s **unknown**, so orders of G^p and G unknown

⇒ **Cannot sample uniformly** from G or G^p!

Sampling exponents

Problem

s **unknown**, so orders of G^p and G unknown

$\Rightarrow$ **Cannot sample uniformly** from G or G^p!

Solution

- Bound on $h(\mathcal{O}_{\Delta_K}) \Rightarrow$ upper bound $\tilde{s}$ for s
- Use $\tilde{s}$ to instantiate distributions $\mathcal{D}$ and $\mathcal{D}_p$ s.t.

$$\{g^x, x \hookleftarrow \mathcal{D}\} \approx \mathcal{U}(G),$$

$$\text{and } \{g_p^x, x \hookleftarrow \mathcal{D}_p\} \approx \mathcal{U}(G^p)$$

- **In practice:** $\mathcal{D}$ and $\mathcal{D}_p$ folded gaussian distributions with large standard deviation.

Linearly Homomorphic Public Key Encryption mod p from HSM

Homomorphic PKE scheme mod p from HSM

KeyGen Sample $t \hookleftarrow \mathcal{D}_p$ and compute $h = g_p^t$

$$\mathsf{sk} = t \text{ and } \mathsf{pk} = h$$

Homomorphic PKE scheme mod p from HSM

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Enc Plaintext: $m \in \mathbf{Z}/p\mathbf{Z}$

Sample randomness $r \hookleftarrow \mathcal{D}_p$

Ciphertext:

$$(C_0, C_1) = (g_p^r, f^m \cdot h^r)$$

Homomorphic PKE scheme mod p from HSM

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Sample randomness $r \hookleftarrow \mathcal{D}_p$

Ciphertext:

$$(C_0, C_1) = (g_p^r, f^m \cdot h^r)$$

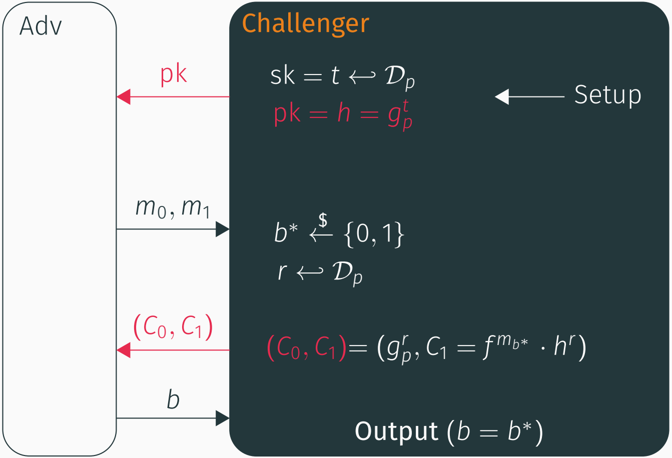
Dec **From** (C_0, C_1) **and** $\mathsf{sk} = t$ **:**

$$C_0/C_1^t \xrightarrow{\text{DL}} m \mod p$$

Security

This scheme is semantically secure under the HSM assumption.

Game 0: the original security experiment



Adv

pk

Challenger

$\mathsf{sk} = t \hookleftarrow \mathcal{D}_p$

$\mathsf{pk} = h = g_p^t$

Setup

m_0, m_1

$b^* \xleftarrow{\$} \{0,1\}$

$r \hookleftarrow \mathcal{D}_p$

(C_0, C_1)

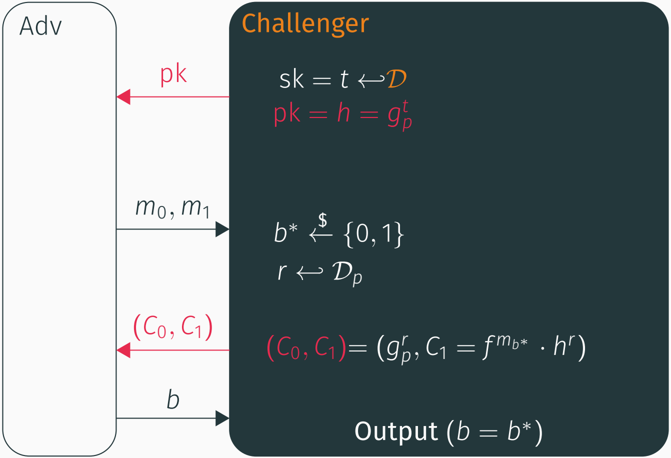
$(C_0, C_1) = (g_p^r, C_1 = f^{m_{b^*}} \cdot h^r)$

b

Output $(b = b^*)$

Game 0 is the original security experiment.

Game 1: sample t from $\mathcal{D}$



Adv

Challenger

pk

$\mathrm{sk} = t \hookleftarrow \mathcal{D}$

$\mathrm{pk} = h = g_p^t$

m_0, m_1

$b^* \overset{\$}{\leftarrow} \{0,1\}$

$r \hookleftarrow \mathcal{D}_p$

(C_0, C_1)

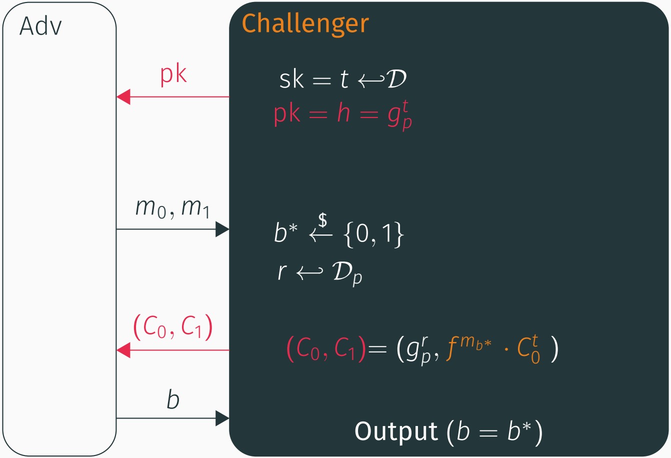
$(C_0, C_1) = (g_p^r, C_1 = f^{m_{b*}} \cdot h^r)$

b

Output $(b = b^*)$

From $\mathcal{A}$'s view, Games 0 and 1 are identical.

Game 2: use sk to compute (C_0, C_1)



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(C_0, C_1)

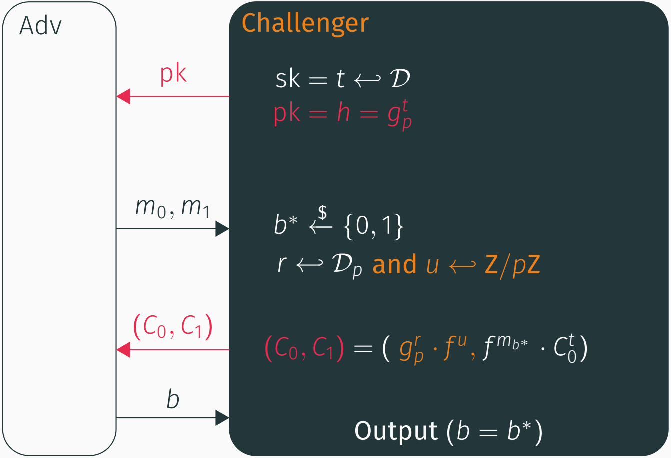
$(C_0, C_1) = (g_p^r, f^{m_{b^*}} \cdot C_0^t)$

b

Output $(b = b^*)$

From $\mathcal{A}$'s view, Games 1 and 2 are identical.

Game 3: compute $C_0 \in G \backslash G^p$



Adv

Challenger

$\mathsf{sk} = t \hookleftarrow \mathcal{D}$

$\mathsf{pk} = h = g_p^t$

pk

m_0, m_1

$b^* \xleftarrow{\$} \{0, 1\}$

$r \hookleftarrow \mathcal{D}_p$ **and** $u \hookleftarrow \mathbf{Z}/p\mathbf{Z}$

(C_0, C_1)

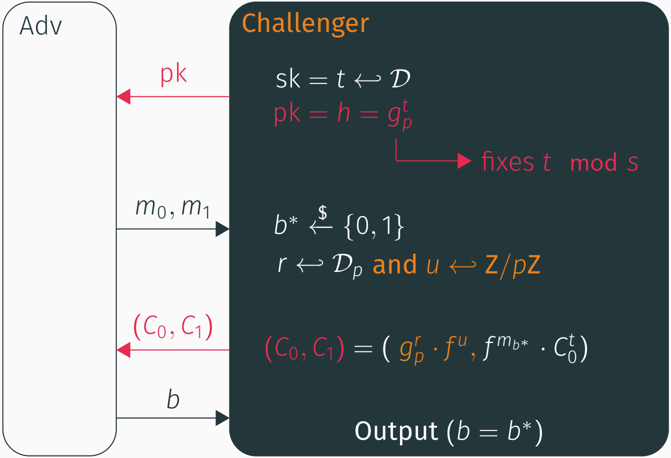
$(C_0, C_1) = (\, g_p^r \cdot f^u, f^{m_{b^*}} \cdot C_0^t)$

b

Output $(b = b^*)$

Games 2 and 3 are undistinguishable to $\mathcal{A}$
under the HSM assumption.

Game 3: compute $C_0 \in G\backslash G^p$



Adv

Challenger

pk

$\mathsf{sk} = t \hookleftarrow \mathcal{D}$

$\mathsf{pk} = h = g_p^t$

fixes $t \mod s$

m_0, m_1

$b^* \xleftarrow{\$} \{0, 1\}$

$r \hookleftarrow \mathcal{D}_p$ and $u \hookleftarrow \mathbf{Z}/p\mathbf{Z}$

(C_0, C_1)

$(C_0, C_1) = (g_p^r \cdot f^u, f^{m_{b^*}} \cdot C_0^t)$

b

Output $(b = b^*)$

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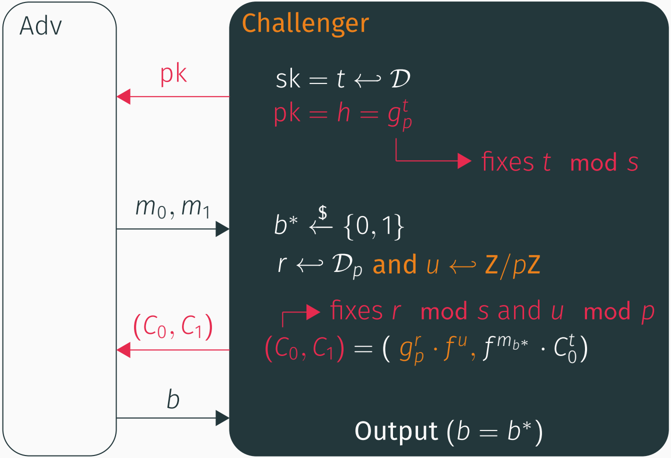

Adv

pk

m_0, m_1

(C_0, C_1)

b

Challenger

$\mathsf{sk} = t \hookleftarrow \mathcal{D}$

$\mathsf{pk} = h = g_p^t$

fixes $t \mod s$

$b^* \xleftarrow{\$} \{0, 1\}$

$r \hookleftarrow \mathcal{D}_p$ **and** $u \hookleftarrow \mathbf{Z}/p\mathbf{Z}$

fixes $r \mod s$ and $u \mod p$

$(C_0, C_1) = (\, g_p^r \cdot f^u, f^{m_{b^*}} \cdot C_0^t)$

Output $(b = b^*)$

Games 2 and 3 are undistinguishable to $\mathcal{A}$
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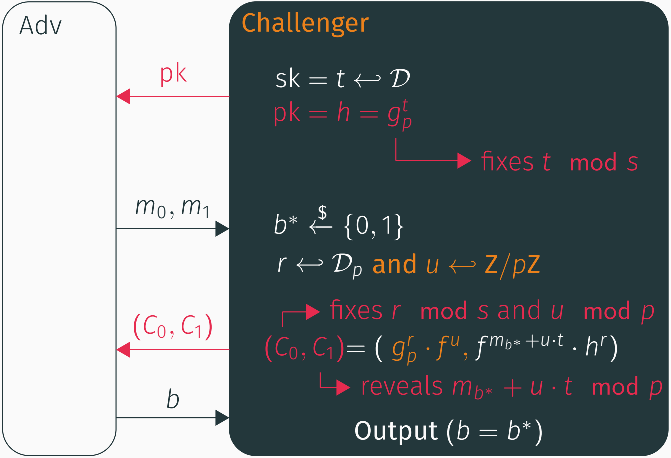
Game 3: compute $C_0 \in G \backslash G^p$



Adv

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fixes $t \mod s$

m_0, m_1

$b^* \xleftarrow{\$} \{0, 1\}$

$r \hookleftarrow \mathcal{D}_p$ **and** $u \hookleftarrow \mathbf{Z}/p\mathbf{Z}$

fixes $r \mod s$ and $u \mod p$

(C_0, C_1)

$(C_0, C_1) = (\, g_p^r \cdot f^u, f^{m_{b^*} + u \cdot t} \cdot h^r)$

reveals $m_{b^*} + u \cdot t \mod p$

b

Output $(b = b^*)$

Games 2 and 3 are undistinguishable to $\mathcal{A}$
under the HSM assumption.

Inner Product Functional Encryption mod p from HSM

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

$\mathsf{msk} = \vec{t}$ and $\mathsf{mpk} = (h_1, \ldots, h_\ell)$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

$\mathsf{msk} = \vec{t}$ and $\mathsf{mpk} = (h_1, \ldots, h_\ell)$

Enc Plaintext: $\vec{y} = (y_1, \ldots, y_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Sample randomness r

Ciphertext:

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_1} \cdot h_1^r, \ldots, C_\ell = f^{y_\ell} \cdot h_\ell^r)$$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

$$\mathsf{msk} = \vec{t} \text{ and } \mathsf{mpk} = (h_1, \ldots, h_\ell)$$

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Ciphertext:

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_1} \cdot h_1^r, \ldots, C_\ell = f^{y_\ell} \cdot h_\ell^r)$$

KeyDer Input: $\vec{x} = (x_1, \ldots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

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Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

Dec From $\vec{C}, \vec{x}$ and $\mathsf{sk}_{\vec{x}}$:

$\langle \vec{x}, \vec{y} \rangle \bmod p$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \dots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \dots, \ell$

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KeyDer **Input:** $\vec{x} = (x_1, \dots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

Dec **From** $\vec{C}, \vec{x}$ **and** $\mathsf{sk}_{\vec{x}}$ **:** $\prod_{i=1}^{\ell} C_i^{x_i}$

$\langle \vec{x}, \vec{y} \rangle \bmod p$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

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KeyDer Input: $\vec{x} = (x_1, \ldots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

Dec From $\vec{C}, \vec{x}$ and $\mathsf{sk}_{\vec{x}}$: $\prod_{i=1}^{\ell} C_i^{x_i} = \prod (f^{\,y_i} \cdot h_i^r)^{x_i}$

$\langle \vec{x}, \vec{y} \rangle$ mod p

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \dots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \dots, \ell$

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Sample randomness r

Ciphertext:

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KeyDer Input: $\vec{x} = (x_1, \dots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

Dec From $\vec{C}, \vec{x}$ and $\mathsf{sk}_{\vec{x}}$: $\prod_{i=1}^{\ell} C_i^{x_i} = f^{\sum y_i x_i} \cdot g_p^{r \cdot \sum t_i x_i}$

$\langle \vec{x}, \vec{y} \rangle \bmod p$

IPFE scheme mod p from HSM (simplified)

<u>**Setup**</u> Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

$\mathsf{msk} = \vec{t}$ and $\mathsf{mpk} = (h_1, \ldots, h_\ell)$

<u>**Enc**</u> **Plaintext:** $\vec{y} = (y_1, \ldots, y_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Sample randomness r

Ciphertext:

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_1} \cdot h_1^r, \ldots, C_\ell = f^{y_\ell} \cdot h_\ell^r)$$

<u>**KeyDer**</u> **Input:** $\vec{x} = (x_1, \ldots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

<u>**Dec**</u> **From** $\vec{C}, \vec{x}$ **and** $\mathsf{sk}_{\vec{x}}$ **:** $\prod_{i=1}^{\ell} C_i^{x_i} = f^{\langle \vec{y}, \vec{x} \rangle} \cdot g_p^{r \cdot \langle \vec{t}, \vec{x} \rangle}$

$\langle \vec{x}, \vec{y} \rangle \bmod p$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \dots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \dots, \ell$

$\mathsf{msk} = \vec{t}$ and $\mathsf{mpk} = (h_1, \dots, h_\ell)$

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Ciphertext:

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_1} \cdot h_1^r, \dots, C_\ell = f^{y_\ell} \cdot h_\ell^r)$$

KeyDer **Input:** $\vec{x} = (x_1, \dots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

Dec **From** $\vec{C}, \vec{x}$ **and** $\mathsf{sk}_{\vec{x}}$ **:** $\prod_{i=1}^{\ell} C_i^{x_i} = f^{\langle \vec{y}, \vec{x} \rangle} \cdot g_p^{r \cdot \langle \vec{t}, \vec{x} \rangle}$ **and** $C_0^{\mathsf{sk}_{\vec{x}}} = g_p^{r \cdot \langle \vec{t}, \vec{x} \rangle}$

$\langle \vec{x}, \vec{y} \rangle \bmod p$

IPFE scheme mod p from HSM (simplified)

Setup Sample $\vec{t} = (t_1, \ldots, t_\ell)$ compute $h_i = g_p^{t_i}$ for $i = 1, \ldots, \ell$

$\mathsf{msk} = \vec{t}$ and $\mathsf{mpk} = (h_1, \ldots, h_\ell)$

Enc **Plaintext:** $\vec{y} = (y_1, \ldots, y_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Sample randomness r

Ciphertext:

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_1} \cdot h_1^r, \ldots, C_\ell = f^{y_\ell} \cdot h_\ell^r)$$

KeyDer **Input:** $\vec{x} = (x_1, \ldots, x_\ell) \in (\mathbf{Z}/p\mathbf{Z})^\ell$

Output key: $\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle$

Dec **From** $\vec{C}, \vec{x}$ **and** $\mathsf{sk}_{\vec{x}}$ **:** $\prod_{i=1}^{\ell} C_i^{x_i} = f^{\langle \vec{y}, \vec{x} \rangle} \cdot g_p^{r \cdot \langle \vec{t}, \vec{x} \rangle}$ **and** $C_0^{\mathsf{sk}_{\vec{x}}} = g_p^{r \cdot \langle \vec{t}, \vec{x} \rangle}$

Such that:

$$\prod_{i=1}^{\ell} C_i^{x_i} / C_0^{\mathsf{sk}_{\vec{x}}} = f^{\langle \vec{x}, \vec{y} \rangle} \xrightarrow{\mathsf{DL}} \langle \vec{x}, \vec{y} \rangle \bmod p$$

Security

This scheme is secure under the HSM assumption.

Proof technique

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_{b*,1}} \cdot h_1^r, \ldots, C_\ell = f^{y_{b*,\ell}} \cdot h_\ell^r)$$

- Game 0 original security game

Proof technique

$$\vec{C} = (C_0 = g_p^r, C_1 = f^{y_{b*,1}} \cdot C_0^{t_1}, \ldots, C_\ell = f^{y_{b*,\ell}} \cdot C_0^{t_\ell})$$

- Game 0 original security game
- Game 1 use **secret key** to compute challenge ciphertext

Proof technique

$$\vec{C} = (C_0 = g_p^r f^u, C_1 = f^{y_{b*,1}} \cdot C_0^{t_1}, \ldots, C_\ell = f^{y_{b*,\ell}} \cdot C_0^{t_\ell})$$

- Game 0 original security game
- Game 1 use **secret key** to compute challenge ciphertext
- Game 2 indistinguishable from Game 1 under the HSM assumption.

Proof technique

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- Game 0 original security game
- Game 1 use **secret key** to compute challenge ciphertext
- Game 2 indistinguishable from Game 1 under the HSM assumption.

In Game 2, from $\mathcal{A}$'s view b^* is **statistically hidden**, given

- the public key
- the challenge ciphertext
- key derivation queries

Efficiency comparison

	$\lambda = 112, \ell = 10$		$\lambda = 128, \ell = 10$	
	this work	[ALS16]	this work	[ALS16]
sk_F bitsize	1920	24592	2340	36876
Enc time	40ms	**27ms**	**78ms**	85ms
Dec time	**110ms**	301ms	**193ms**	964ms

Dependency in ℓ is linear.

Last slide!

Conclusion

- Most efficient IPFE schemes to date
- First IPFE mod a prime that recover the result whatever its size.
- Interesting framework, can be applied to other primitives.

Ongoing work

- Chosen Ciphertext Attack Secure schemes
- Threshold ECDSA using our underlying framework

Questions?

M. Abdalla, F. Bourse, A. D. Caro, and D. Pointcheval.
Better security for functional encryption for inner product evaluations.
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S. Agrawal, B. Libert, and D. Stehlé.
Fully secure functional encryption for inner products, from standard assumptions.
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F. Benhamouda, F. Bourse, and H. Lipmaa.
CCA-secure inner-product functional encryption from projective hash functions.
In *PKC 2017, Part II*, *LNCS* 10175, pages 36–66. Springer, Heidelberg, March 2017.

Information $\mathcal{A}$ gets on b^* in PKE

$$m_{b^*} + u \cdot t \mod p$$

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Where:

(1) $u \neq 0 \mod p$ with proba $\frac{p-1}{p} \approx 1$

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Where:

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and

(2) t sampled from $\mathcal{D}$, folded gaussian, (almost) uniform mod $s \cdot p$

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and

(2) t sampled from $\mathcal{D}$, folded gaussian, (almost) uniform mod $s \cdot p$

↓

Distribution of t (almost) uniform mod p and mod s
and $(t \mod p)$ independent of $(t \mod s)$

Information $\mathcal{A}$ gets on b^* in PKE

$$m_{b*} + u \cdot t \mod p$$

Where:

(1) $u \neq 0 \mod p$ with proba $\frac{p-1}{p} \approx 1$

and

(2) t sampled from $\mathcal{D}$, folded gaussian, (almost) uniform mod $s \cdot p$

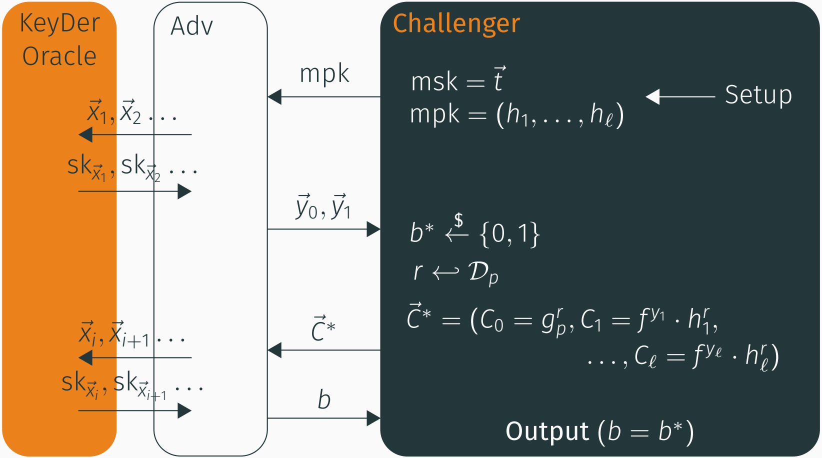
$\downarrow$

Distribution of t (almost) uniform mod p and mod s
and $(t \mod p)$ independent of $(t \mod s)$

$\downarrow$

$u \cdot t$ perfectly masks m_{b*} mod p

Game 0: the original security experiment



KeyDer Oracle

Adv

Challenger

mpk

$\mathsf{msk} = \vec{t}$

$\mathsf{mpk} = (h_1, \ldots, h_\ell)$

Setup

$\vec{x}_1, \vec{x}_2 \ldots$

$\mathsf{sk}_{\vec{x}_1}, \mathsf{sk}_{\vec{x}_2} \ldots$

$\vec{y}_0, \vec{y}_1$

$b^* \xleftarrow{\$} \{0, 1\}$

$r \hookleftarrow \mathcal{D}_p$

$\vec{C}^* = (C_0 = g_p^r, C_1 = f^{y_1} \cdot h_1^r,$

$\ldots, C_\ell = f^{y_\ell} \cdot h_\ell^r)$

$\vec{x}_i, \vec{x}_{i+1} \ldots$

$\vec{C}^*$

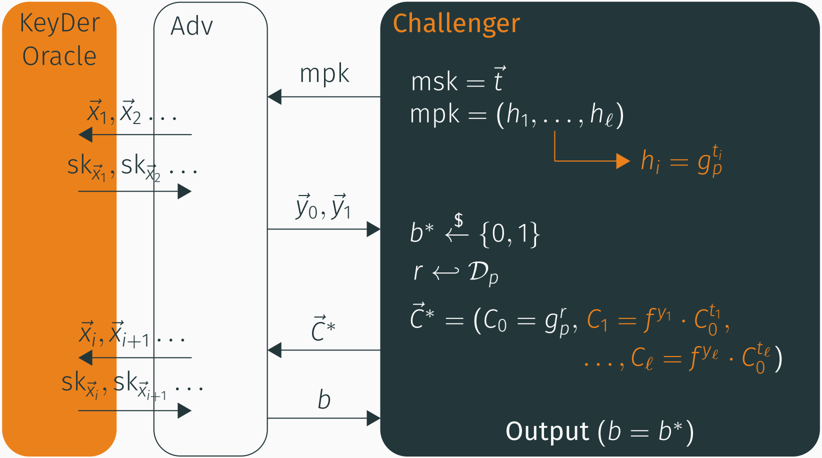
$\mathsf{sk}_{\vec{x}_i}, \mathsf{sk}_{\vec{x}_{i+1}} \ldots$

b

Output $(b = b^*)$

Game 0 is the original security experiment.

Game 1: use msk to compute $\vec{C}^*$



KeyDer Oracle

Adv

Challenger

$\vec{x}_1, \vec{x}_2 \ldots$

$\mathsf{sk}_{\vec{x}_1}, \mathsf{sk}_{\vec{x}_2} \ldots$

$\vec{x}_i, \vec{x}_{i+1} \ldots$

$\mathsf{sk}_{\vec{x}_i}, \mathsf{sk}_{\vec{x}_{i+1}} \ldots$

mpk

$\vec{y}_0, \vec{y}_1$

$\vec{C}^*$

b

$\mathsf{msk} = \vec{t}$

$\mathsf{mpk} = (h_1, \ldots, h_\ell)$

$h_i = g_p^{t_i}$

$b^* \overset{\$}{\leftarrow} \{0,1\}$

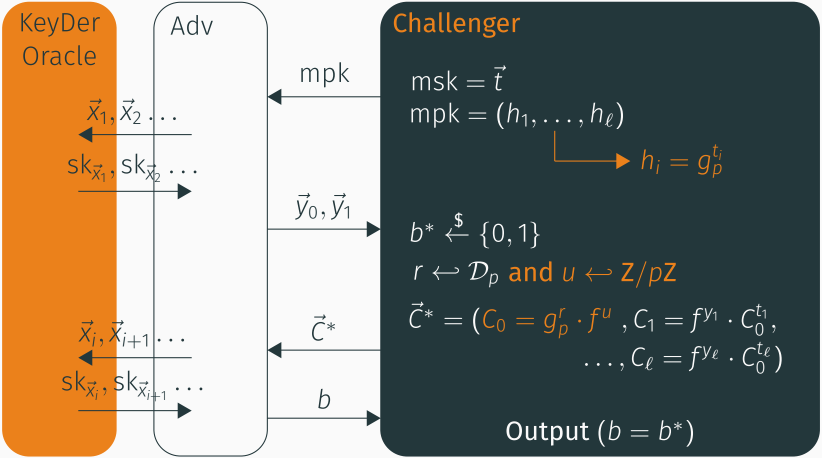
$r \hookleftarrow \mathcal{D}_p$

$$\vec{C}^* = (C_0 = g_p^r, C_1 = f^{y_1} \cdot C_0^{t_1}, \ldots, C_\ell = f^{y_\ell} \cdot C_0^{t_\ell})$$

Output $(b = b^*)$

From $\mathcal{A}$'s view, Games 0 and 1 are identical.

Game 2: compute $C_0 \in G \backslash G^p$



KeyDer Oracle

Adv

Challenger

mpk

$\mathsf{msk} = \vec{t}$

$\mathsf{mpk} = (h_1, \ldots, h_\ell)$

$h_i = g_p^{t_i}$

$\vec{x}_1, \vec{x}_2 \ldots$

$\mathsf{sk}_{\vec{x}_1}, \mathsf{sk}_{\vec{x}_2} \ldots$

$\vec{y}_0, \vec{y}_1$

$b^* \xleftarrow{\$} \{0,1\}$

$r \hookleftarrow \mathcal{D}_p$ **and** $u \hookleftarrow \mathbf{Z}/p\mathbf{Z}$

$\vec{C}^*$

$$\vec{C}^* = (C_0 = g_p^r \cdot f^u, C_1 = f^{y_1} \cdot C_0^{t_1}, \ldots, C_\ell = f^{y_\ell} \cdot C_0^{t_\ell})$$

$\vec{x}_i, \vec{x}_{i+1} \ldots$

$\mathsf{sk}_{\vec{x}_i}, \mathsf{sk}_{\vec{x}_{i+1}} \ldots$

b

Output $(b = b^*)$

Games 1 and 2 are undistinguishable to $\mathcal{A}$ under the HSM assumption.

Leaked Information in Game 2

We consider the information leaked on b^* by:

- the public key
- the challenge ciphertext
- key derivation queries

Information fixed by public key

$$\mathsf{mpk} = \{h_i = g_p^{t_i \bmod s}\}_{i \in [\ell]}$$

Information fixed by public key

$$\begin{tikzcd}
\mathsf{mpk} = \{h_i = g_p^{t_i \bmod s}\}_{i \in [\ell]} \arrow[d, "\text{Fixes}"] \\
(t_1, \ldots, t_\ell) \bmod s
\end{tikzcd}$$

Information fixed by public key

$$\mathsf{mpk} = \{h_i = g_p^{t_i \bmod s}\}_{i \in [\ell]}$$

Fixes

$\downarrow$

$(t_1, \ldots, t_\ell) \bmod s$

$(t_1, \ldots, t_\ell) \bmod p$ is still **uniformly** distributed to $\mathcal{A}$.

Information fixed by challenge ciphertext

$$\vec{C}^* = (C_0 = g_p^r \cdot f^u, \{C_i = f^{y_{b^*,i}} \cdot C_0{}^{t_i}\}_{i \in [\ell]})$$

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$$\begin{tikzcd}
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C_i = g_p^{r \cdot t_i \mod s} \cdot f^{y_{b*,i} + u \cdot t_i \mod p}
\end{tikzcd}$$

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Reveals

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Fixes

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Information fixed by key derivation oracle

For $\vec{x}$ such that $\langle \vec{x}, \vec{y}_0 \rangle = \langle \vec{x}, \vec{y}_1 \rangle \mod p$:

$$\mathsf{sk}_{\vec{x}} = \langle \vec{t}, \vec{x} \rangle \mod p$$

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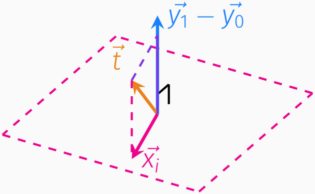

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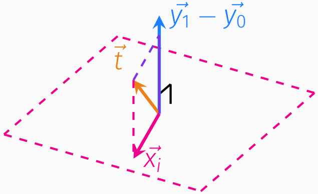

Remaining entropy on $\vec{t}$ contained in $\langle \vec{t}, \vec{y_0} - \vec{y_1} \rangle$

$\mathcal{A}$'s success probability

From $\mathcal{A}$'s view, $\langle \vec{t}, \vec{y}_0 - \vec{y}_1 \rangle$ follows a distribution $\approx \mathcal{U}(\mathsf{Z}/p\mathsf{Z})$.

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$\mathcal{A}$ cannot guess b^* with proba $> 1/2 +$ negl