CM-Points on Straight Lines

Yuri Bilu

A joint work with Bill Allombert and Amalia Pizarro-Madariaga

Let τ be an imaginary quadratic number with $\mathrm{Im}\tau > 0$ and let j denote the j-invariant function. According to the classical theory of Complex Multiplication, the complex number $j(\tau)$ is an algebraic integer. A *CM-point* in $\mathbb{C}^2$ is a point of the form $(j(\tau_1), j(\tau_2))$, where both τ_1 and τ_2 are imaginary quadratic numbers.

In 1998 Yves André proved that a non-special (the notion will be defined during the talk) irreducible plane curve $F(x_1, x_2) = 0$ may have only finitely many CM-points. This was the first non-trivial contribution to the celebrated André-Oort conjecture.

Relying on recent ideas of Lars Kühne, we obtain a very explicit version of this result for straight lines defined over $\mathbb{Q}$: with "obvious" exceptions, a CM-point cannot belong to such a line. Kühne himself proved this for the line $x_1 + x_2 = 1$.